Utilisation des données à priori des fournisseurs pour une gestion efficiente des résultats de CIQ: les atouts de la logique Bayésienne vs l'approche conventionnelle

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Frequentist versus Bayesian Approach in Statistics

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- Goodness of fit tests and plots should be used to examine how well an assumed theoretical distribution fits the observed data.

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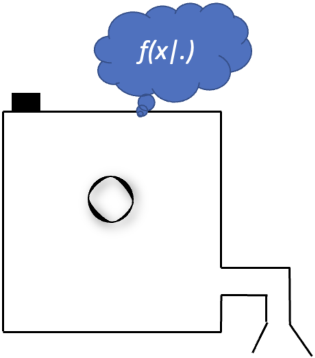

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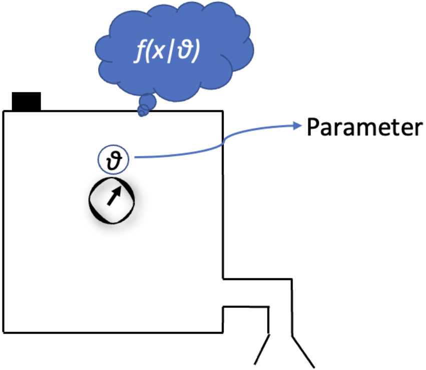

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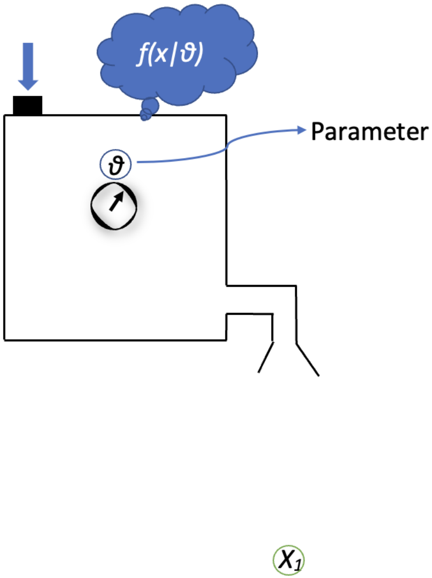

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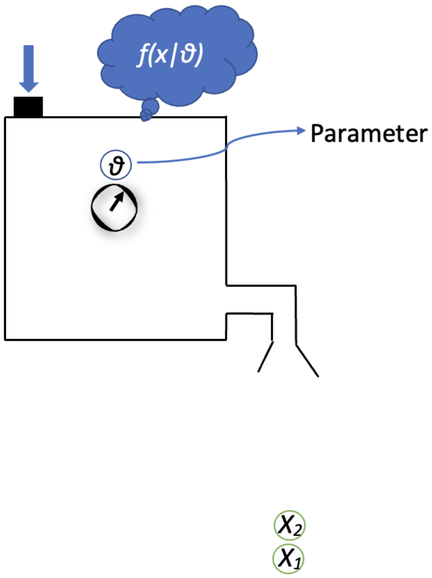

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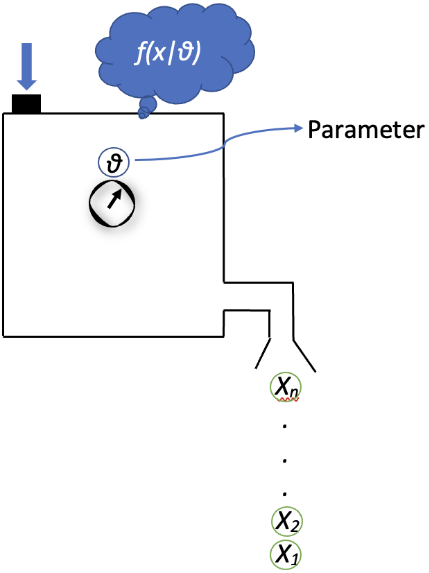

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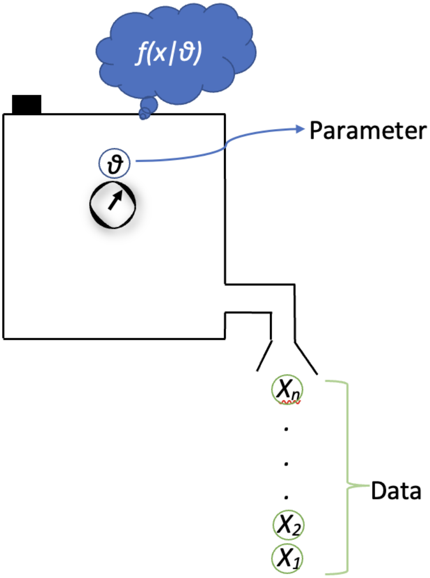

Treating the unknown parameter θ

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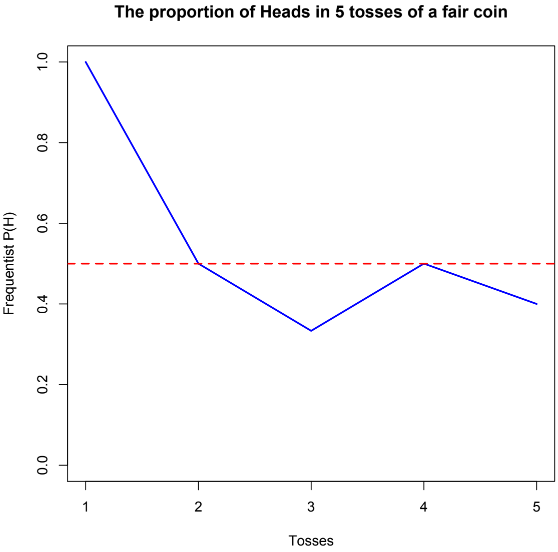
- **Example:** In the case of a fair coin toss, the frequentist probability $P(H)$ is $1/2$, not because there are two equally likely outcomes (classical interpretation of probability) but because in repeated trials the empirical frequency converges to the limit $1/2$ as the number of trials goes to infinity, i.e.

$$P(H) = \frac{\#\text{ of heads}}{\#\text{ of trials}} = \frac{n_H}{n} \xrightarrow{n \to \infty} \frac{1}{2}$$

Frequentist School: probability interpretation



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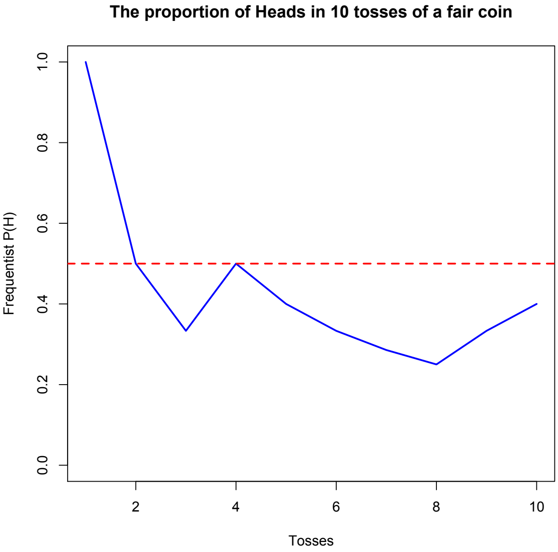

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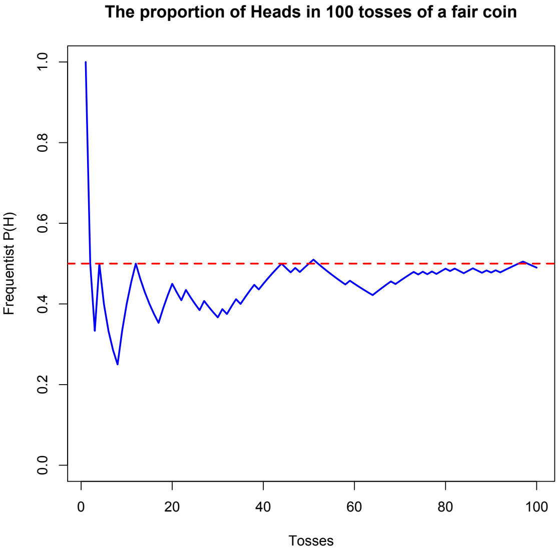

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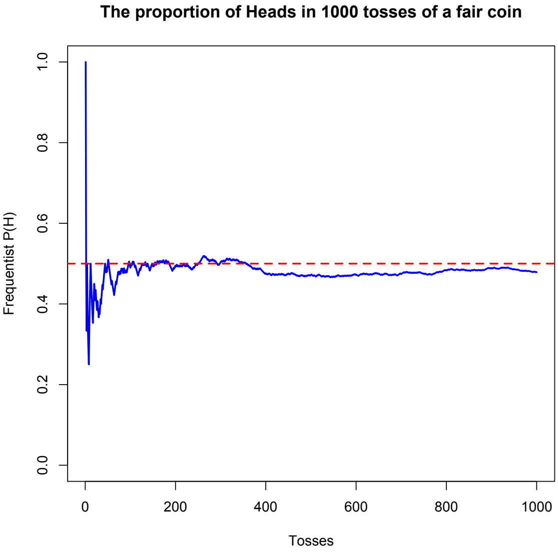

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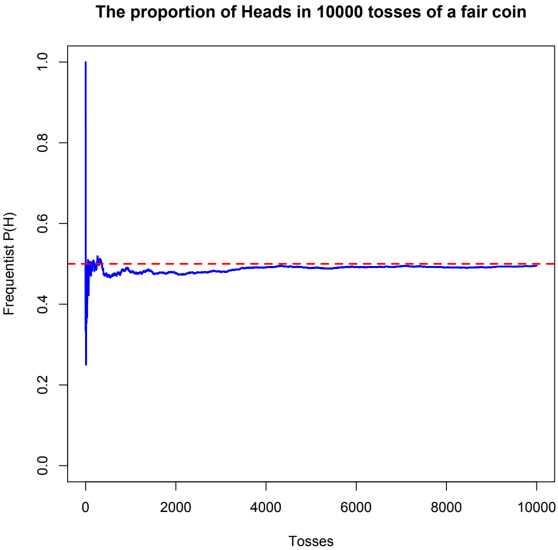

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  - **predictions** for future observable(s).

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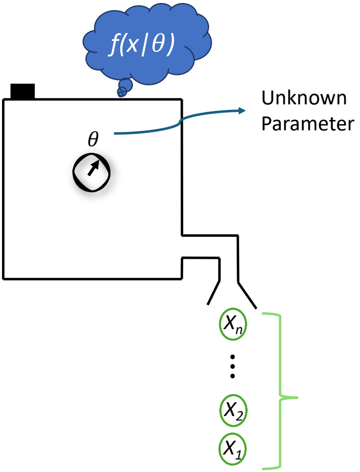

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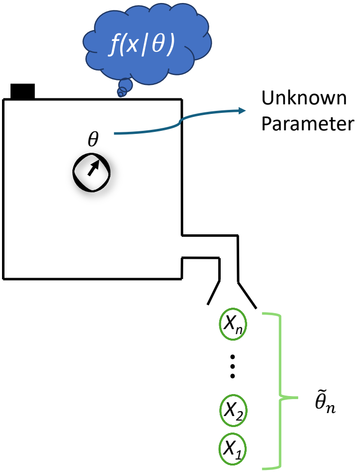

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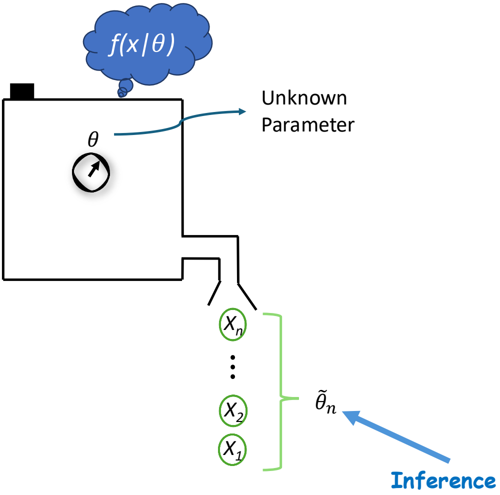

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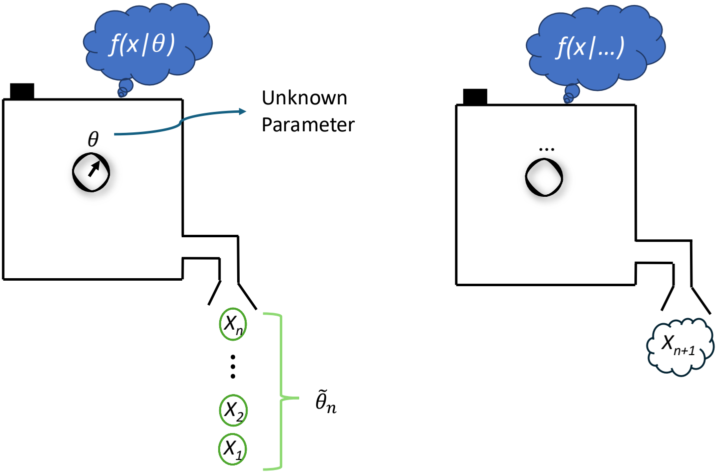

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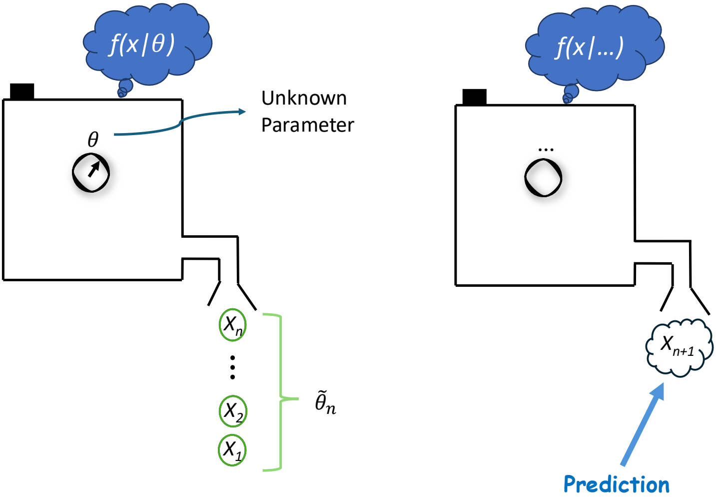

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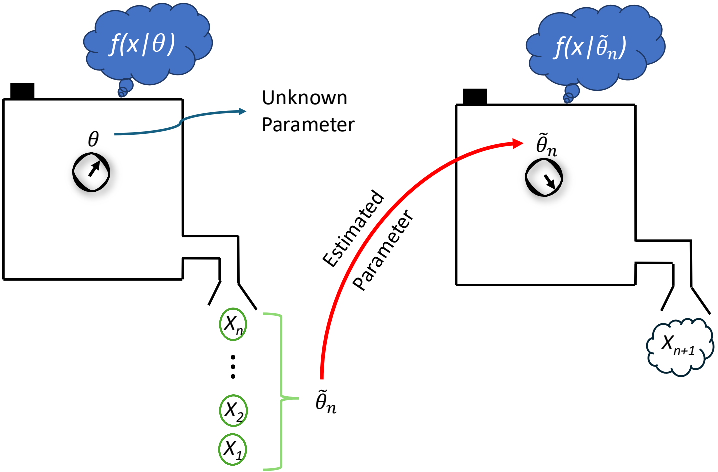

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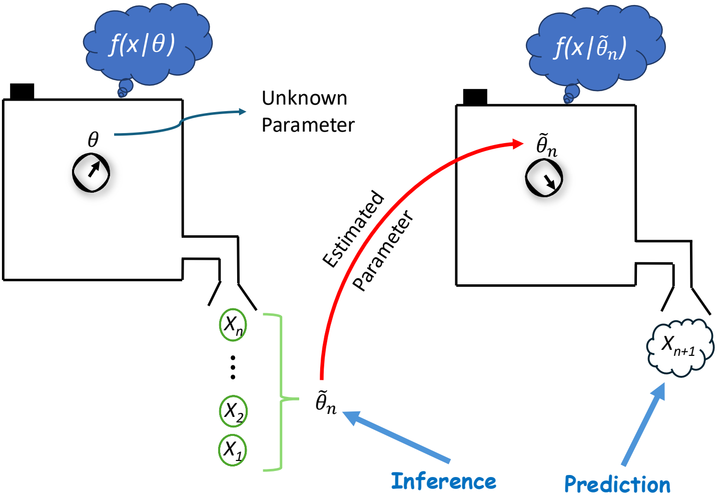

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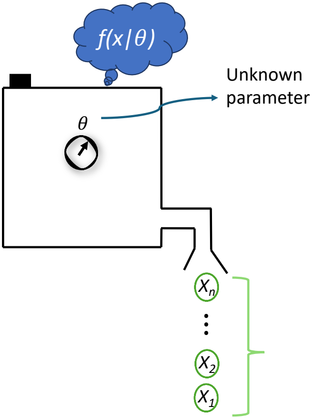

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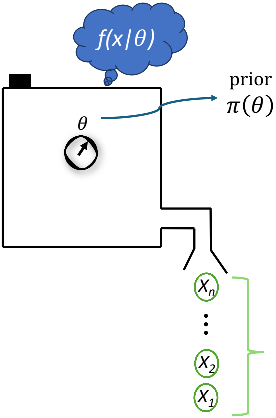

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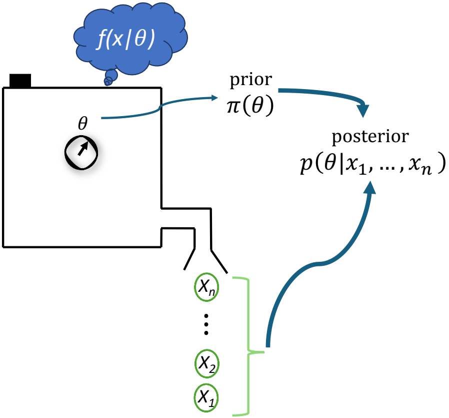

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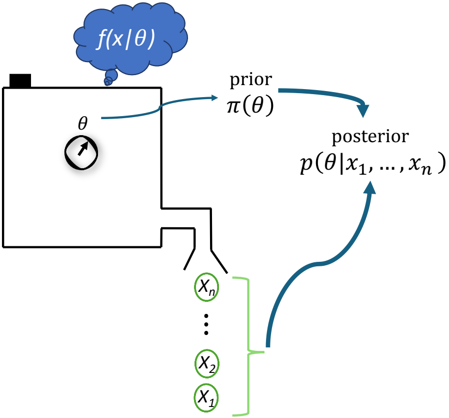

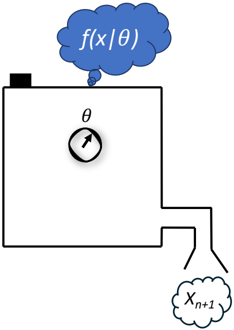

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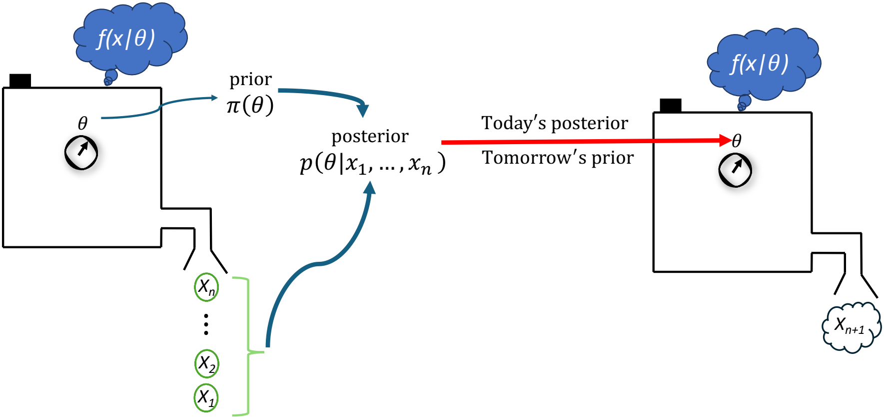

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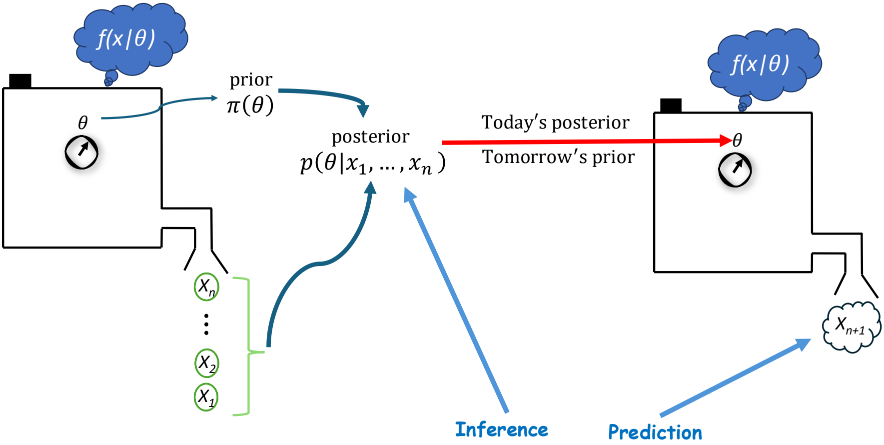

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where:

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  - $f(\mathbf{x}|\theta)$ refers to the likelihood of the data
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- So the Bayes theorem is nothing more that an **updating mechanism**, where the prior is updated to posterior in the light of evidence coming from the available data.

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(e) Derive the predictive distribution $f(x_{n+1}|x_1, x_2, \ldots, x_n)$ of a future observable and make predictions.

Internal & External Bayesian Quality Control

Bayesian Statistical Process Control/Monitoring (SPC/M)

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- **What is Statistical Process Control & Monitoring (SPC/M)?**
  SPC/M is a method of internal/external quality control, which uses a statistical approach (**control charts**) to monitor and control a process.

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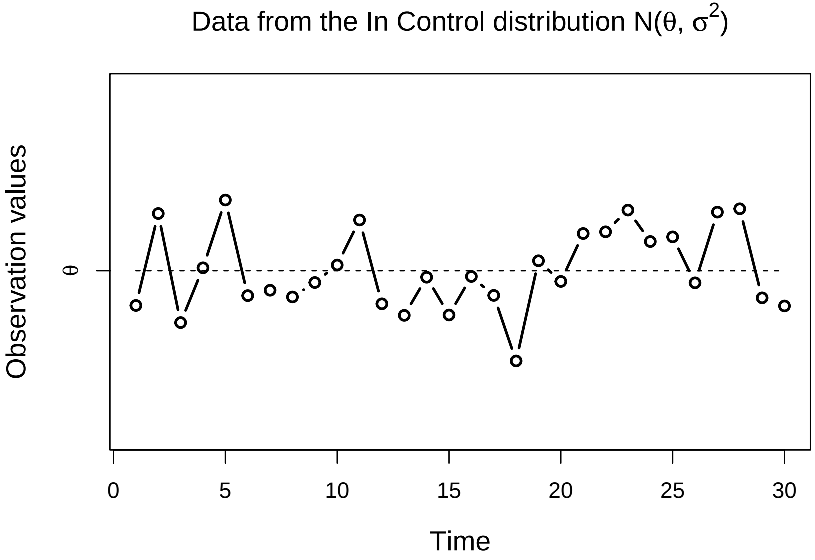

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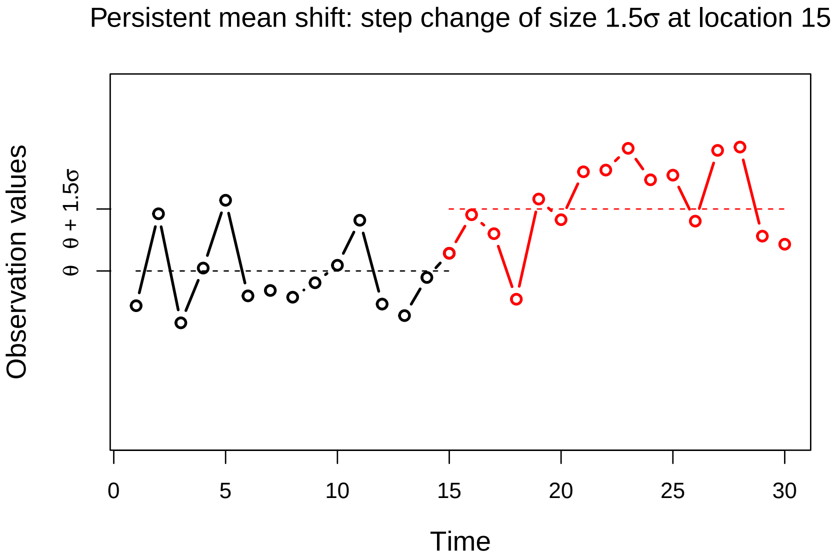

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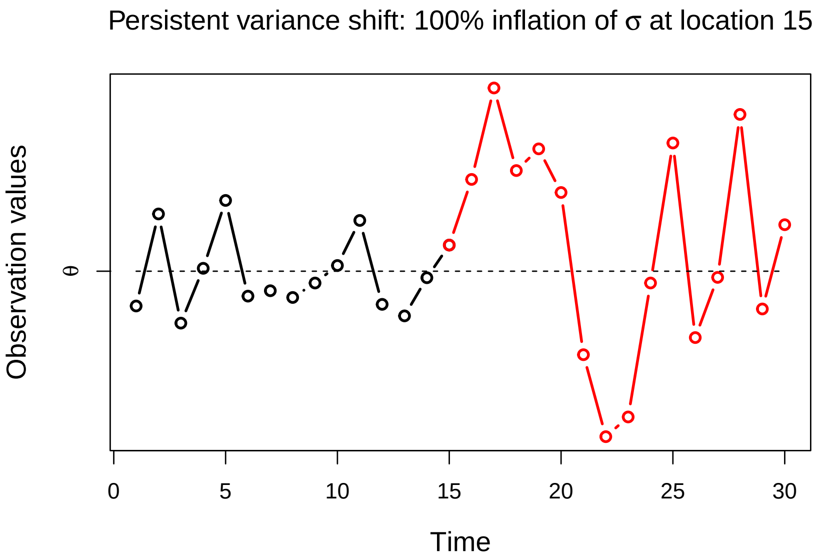

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- This approach is reasonable as long as all the data are IC. OOC points during calibration, will result **contaminated** parameter estimates.
- To anticipate the such risks people tend to use simultaneously several control charts and/or several run rules aiming to increase the power of detecting OOC scenarios. However, **there is no free lunch**! The **more** the charts/rules you combine the **higher** the **false alarm rate**.

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- Prior information (typically available) regarding $\boldsymbol{\theta}$ is left unexploited.

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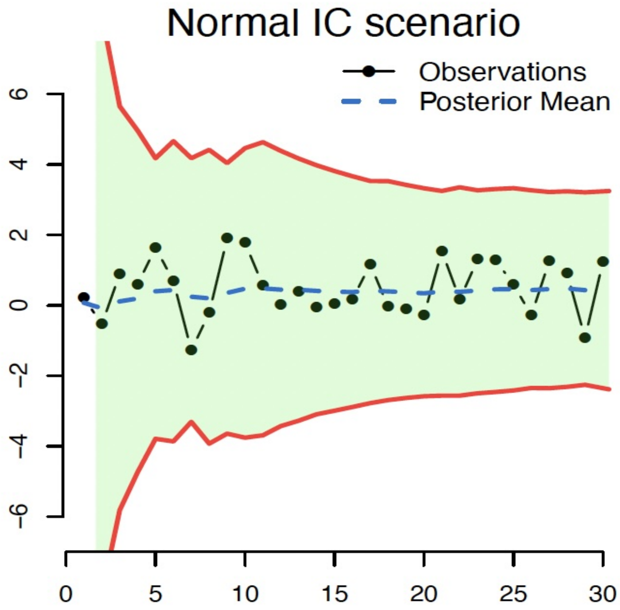
PRC: Predictive Residual Cusum: for detection of **persistent** shifts of medium/small size (extended to Predictive Ratio Cusum for any distribution in the exponential family).

PCC Illustration and Decision Making



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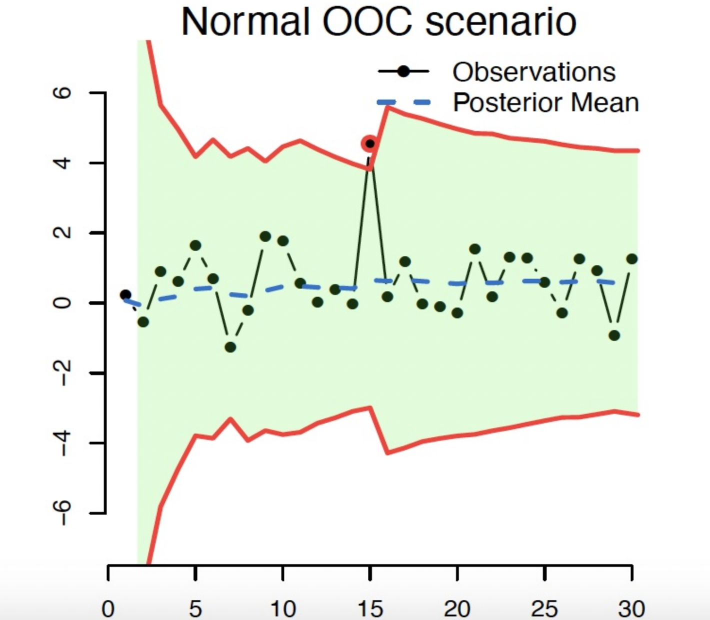
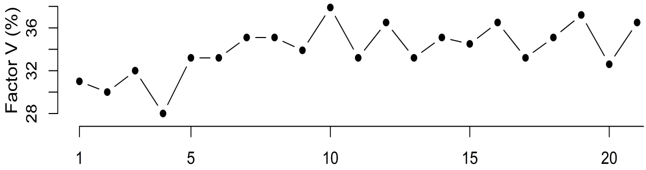
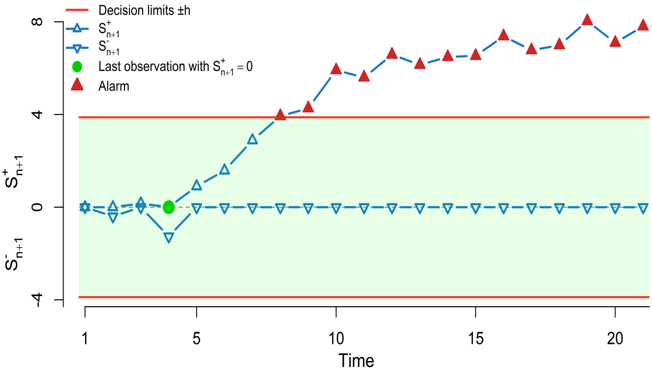


PRC Illustration and Decision Making

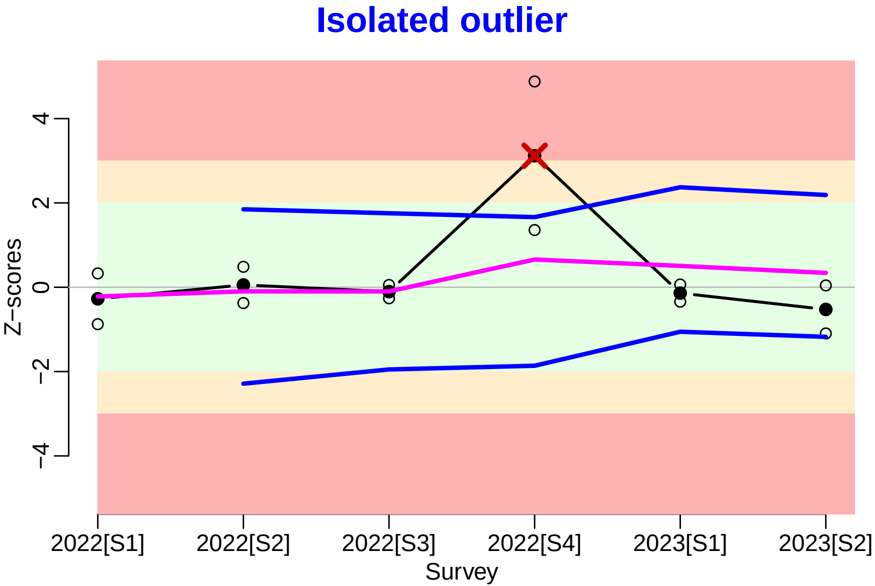




PCC in ECAT's z-score historic evaluation

- ECAT's EQA program evaluates the deviation of the measured result from the assigned value.
- The performance is statistically quantified using Z-scores.
- Univariate and bivariate z-score analysis provides feedback for the current state of the lab.
- In addition, ECAT provides the recent history of the lab's z-scores to help them evaluate their longitudinal performance.

PCC Illustration on ECAT's z-score history



Conclusions

- In Bayesian SPC/M we introduced the **Predictive Control Chart** (**PCC**) and the **Predictive Ratio Cusum** (**PRC**) mechanisms which:
  - can be used in IQC and EQA monitoring processes
  - they utilize available **prior information** and/or **historical data**, boosting the performance.
  - They can identify **outlier** & **change point** problems in the process.
  - They can provide **posterior inference** for the unknown parameter(s) is also available.
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- With low volumes of data, standard statistics might be in trouble!
- Modern alternatives call for utilizing available prior information, opening widely the door to the **Bayesian** approach!

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- This work would not have been existed without **Frederic Sobas** from Hospices Civils de Lyon, France, who provided an actual problem setting that inspired this work, he supported us with different ways, but most importantly he provided invaluable feedback from using **PCC** & **PRC** at the daily Internal Quality Control routine in the medical labs of Hospices Civils de Lyon.

Bayesian SPC/M: Open Access manuscripts

- Bourazas, K., Kiagias, D., and Tsiamyrtzis, P. (2022). **"Predictive Control Charts (PCC): A Bayesian approach in online monitoring of short runs"**. Journal of Quality Technology, Vol. 54 (4):367–391. https://doi.org/10.1080/00224065.2021.1916413

- Bourazas K., Sobas F. and Tsiamyrtzis, P. (2023). **"Predictive ratio CUSUM (PRC): A Bayesian approach in online change point detection of short runs"**. Journal of Quality Technology, Vol. 55(4):391-403. https://doi.org/10.1080/00224065.2022.2161434

- Bourazas K., Sobas F. and Tsiamyrtzis, P. (2023) **"Design and properties of the predictive ratio cusum (PRC) control charts"**. Journal of Quality Technology, Vol. 55(4):404-421. https://doi.org/10.1080/00224065.2022.2161435

PRC paper won the ASQ's 2024 Brumbaugh Award

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Year	Recipient	Winning Article
2024	Konstantinos Bourazas, Frédéric Sobas, Panagiotis Tsiamyrtzis	Predictive ratio CUSUM (PRC): A Bayesian approach in online change point detection of short runs, Journal of Quality Technology, 55:4, 391-403.

Thomas Bayes

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